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Kolstad: Hotelling with Two Stocks AND LINEAR Array of Demanders

Linear Geography
Demanders in uniform density (= unity)
(Quantity defined by distance)

A homogeneous stock at each end.
Each stock typically has a distinct unit extraction cost

c^L on left usually exceeds c^R on right.

Transportation or delivery costs constant per unit distance
(same from either end)

Same Discount rate used for each deposit.

Competitive extraction - each ton owned by a distinct firm.

Rents Left and Right rise at r %.

Phase I
market is covered (fully supplied) by sum of quantities from Left and Right at each instant.

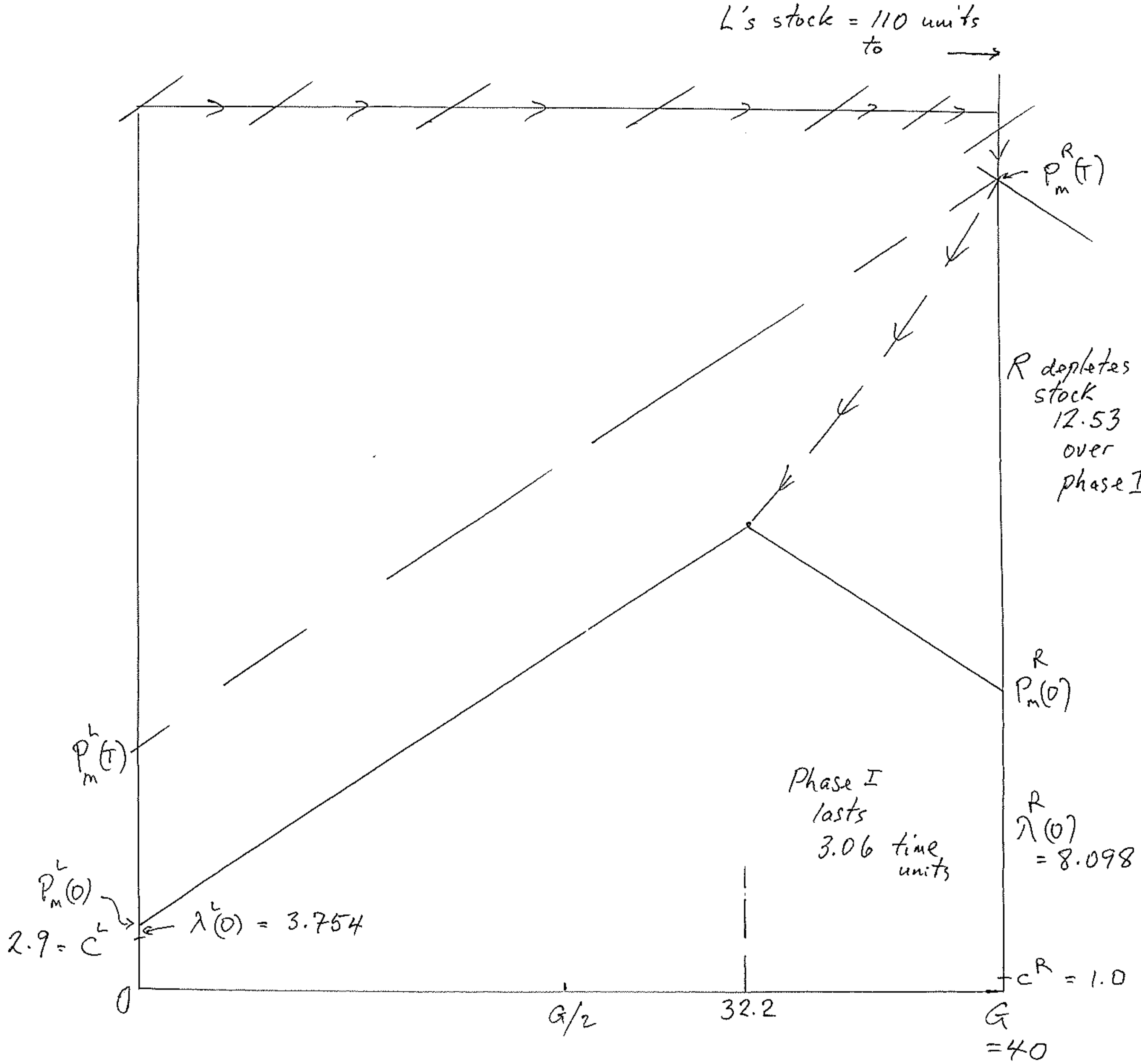
Phase II: a HOLE opens up in the "center" of the market and each remaining stock gets exhausted "independently".

Solution Strategy: Solve BACKWARDS
Employ r % Rule on Rent motion.

discount rate = 0.1
transport cost = 0.1

2, 3, 4

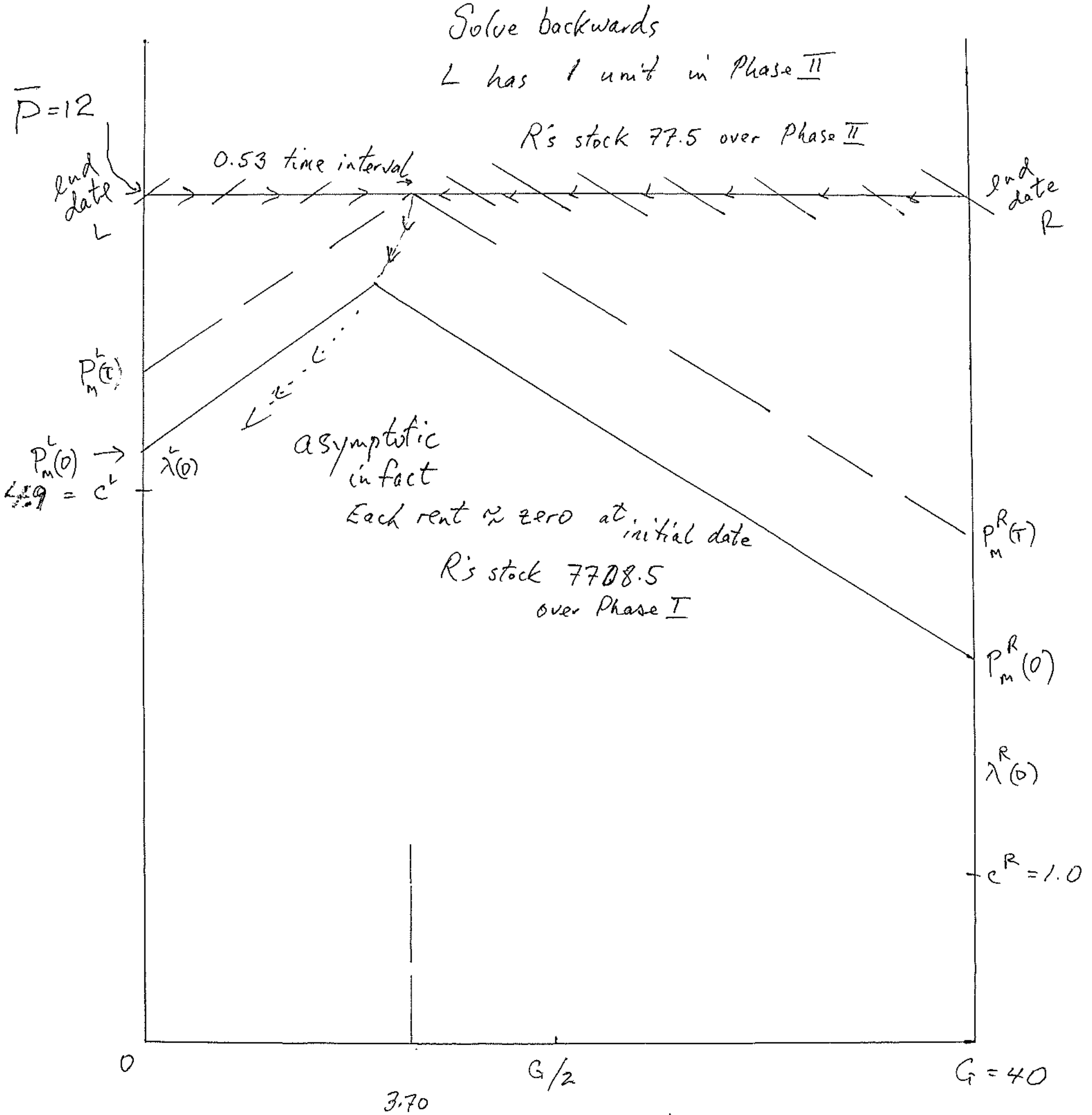
Solve backwards



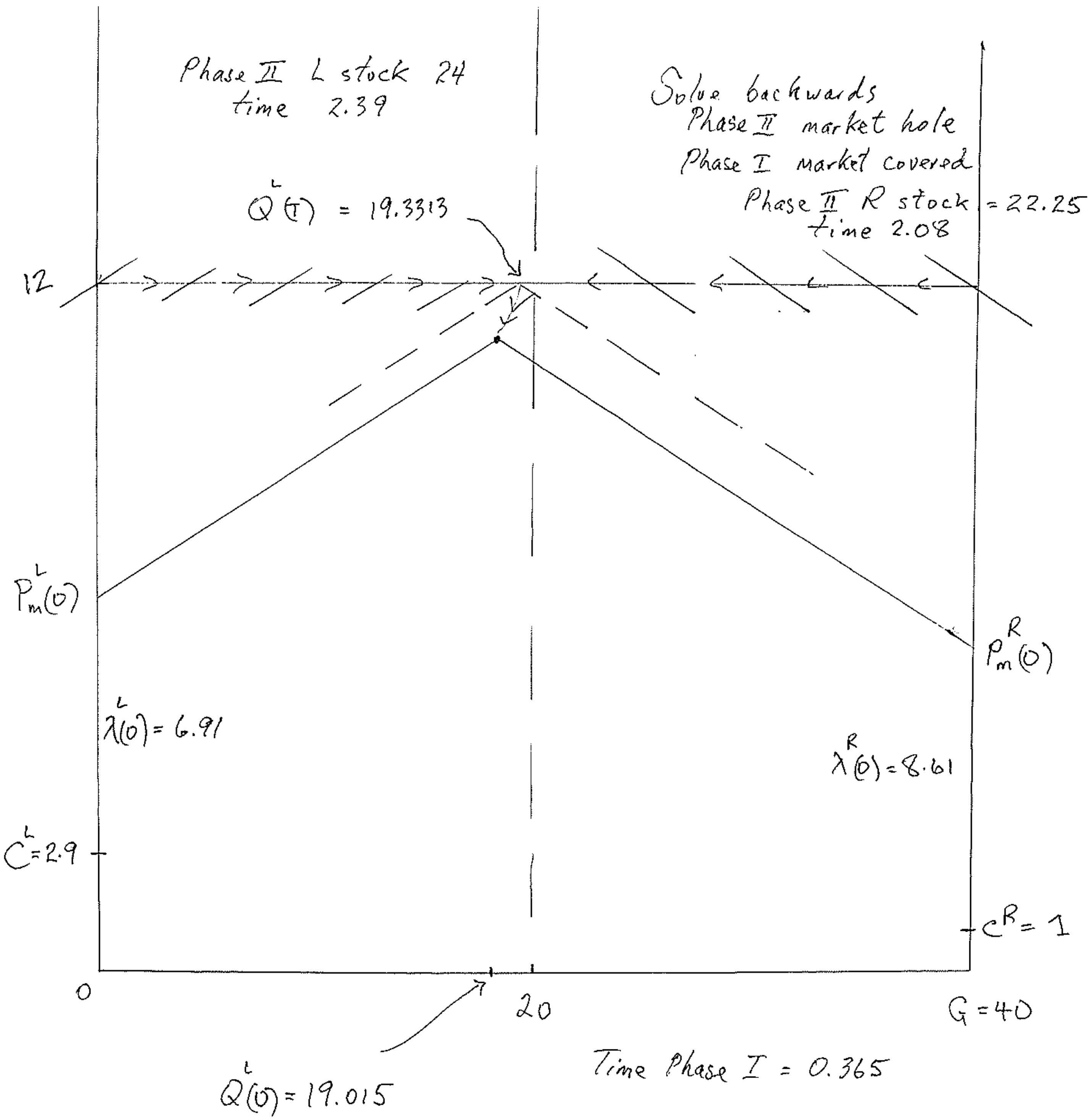
Low Cost has small stock

Kolstad (1994)

discount rate = 0.1
transport cost = 0.1



High cost with Small Stock for L



L uses up 7 units in Phase I

R uses up 7.605 units in Phase I

transport cost 0.1 per unit distance

discount rate = 0.1

ANALYSIS STRATEGY (illustrate similar stock sizes + very distinct costs)

$\tilde{S}^L$ is stock used up in Phase II by L

$\tilde{T}^L$ is length of L's Phase II

Rent shrinks at r% over Phase II for L and R.

$$[\bar{p} - c^L] e^{-r[\tilde{T}^L - t]} \quad \text{over } L\text{'s Phase II}$$

Also

$$c^L + \lambda^L(t) + \alpha Q^L(t) = \bar{p} \quad \text{over } L\text{'s Phase II}.$$

Combine to get

$$Q^L(t) = \frac{\bar{p} - c^L}{\alpha} \left[1 - e^{-r[\tilde{T}^L - t]}\right].$$

Given $\tilde{S}^L$, we have

$$\int_0^{\tilde{T}^L} Q^L(t)\,dt = \tilde{S}^L.$$

Insert expression for $Q^L(t)$ and integrate. Get

$$\tilde{S}^L = \frac{\bar{p} - c^L}{\alpha} \left[\tilde{T}^L + \frac{1}{r}\left(e^{-r\tilde{T}^L} - 1\right)\right]$$

For $G = 40$, $\alpha = 0.1$, $c^R = 1$, $c^L = 2.9$, $\tilde{S}^L = 24$, $\bar{p} = 31$

get

$$\left.\begin{aligned} \tilde{T}^L &= 2.38806 \\ Q^L_T &= 19.33133 \end{aligned}\right\} \text{numerical solve}$$

This Q^L_T gives $Q^R_T = G - Q^L_T$ or "space" R occupies in Phase II.

R's Phase II

$$\lambda^R(T) = \left[\bar{p} - \alpha\left[G - Q_T^L\right] - c^R\right]$$

Discounted terminal rent is $[\bar{p} - c^R]e^{-r\tilde{T}^R}$

Substitute for G and Q_T^L from above

$$\tilde{T}^R = -\frac{1}{r}\ln\left\{\frac{\bar{p} - \alpha[G - Q_T^L] - c^R}{[\bar{p} - c^R]}\right\}$$

yields

$$\tilde{T}^R = 2.08128$$

Phase II stock used up by R is endogenous here.

$$\tilde{S}^R = \frac{\bar{p} - c^R}{\alpha}\left\{\tilde{T}^R + \frac{1}{r}\left(e^{-r\tilde{T}^R} - 1\right)\right\} = 22.2542 \text{ tons}$$

Moving back in time to Phase I for L.

L's rent is now

$$\lambda^L(t) = \left[\bar{p} - \alpha Q_T^L - c^L\right]e^{-r[T-t]}$$

Delivered price same at "center" for L and R

$$c^L + \lambda^L(t) + \alpha Q^L(t) = c^R + \lambda^R(t) + \alpha\left[G - Q^L(t)\right].$$

R's rent is $\lambda^R(t) = \left[\bar{p} - \alpha(G - Q_T^L) - c^R\right]e^{-r[T-t]}$

We solve for

$$Q^L(t) = \frac{1}{2\alpha}\left\{\left[c^R + \alpha G - c^L\right]\left\{1 - e^{-r[T-t]}\right\} + 2\alpha Q_T^L e^{-r[T-t]}\right\}$$

$$S_0^L = K^L - \tilde{S}^L = \int_0^T Q^L(t)\,dt.$$

$$S_0^L = \left[\frac{c^R + \alpha G - c^L}{2\alpha}\right] T - \left(1 - e^{-rT}\right)\left[\frac{c^R + \alpha G - c^L - 2\alpha Q_T^L}{2\alpha r}\right]$$

For $S_0^L = 7$

$T = 0.36511$ (since $T \cdot G = S_0^L + S_0^R$, we have S_0^R to work with)

$$K^R = \tilde{S}^R + S_0^R = 22.2542 + 7.6046$$

Each of $\tilde{S}_R$ and S_0^R has been worked with as endogenous.

$$\lambda^L(0) = \left[\bar{p} - \alpha Q_T^L - c^L\right] e^{-0.36511r} = 6.9099$$

$$\lambda^R(0) = \left[\bar{p} - \alpha\left[G - Q_T^L\right] - c^R\right] e^{-0.36511r} = 8.6129$$

L's initial quantity is 19.0147

and R's is $40 - 19.0147 = 20.9853$

Point of market split moves to Right in Phase I.

"COUNTER INTUITIVE"
RENT CASE

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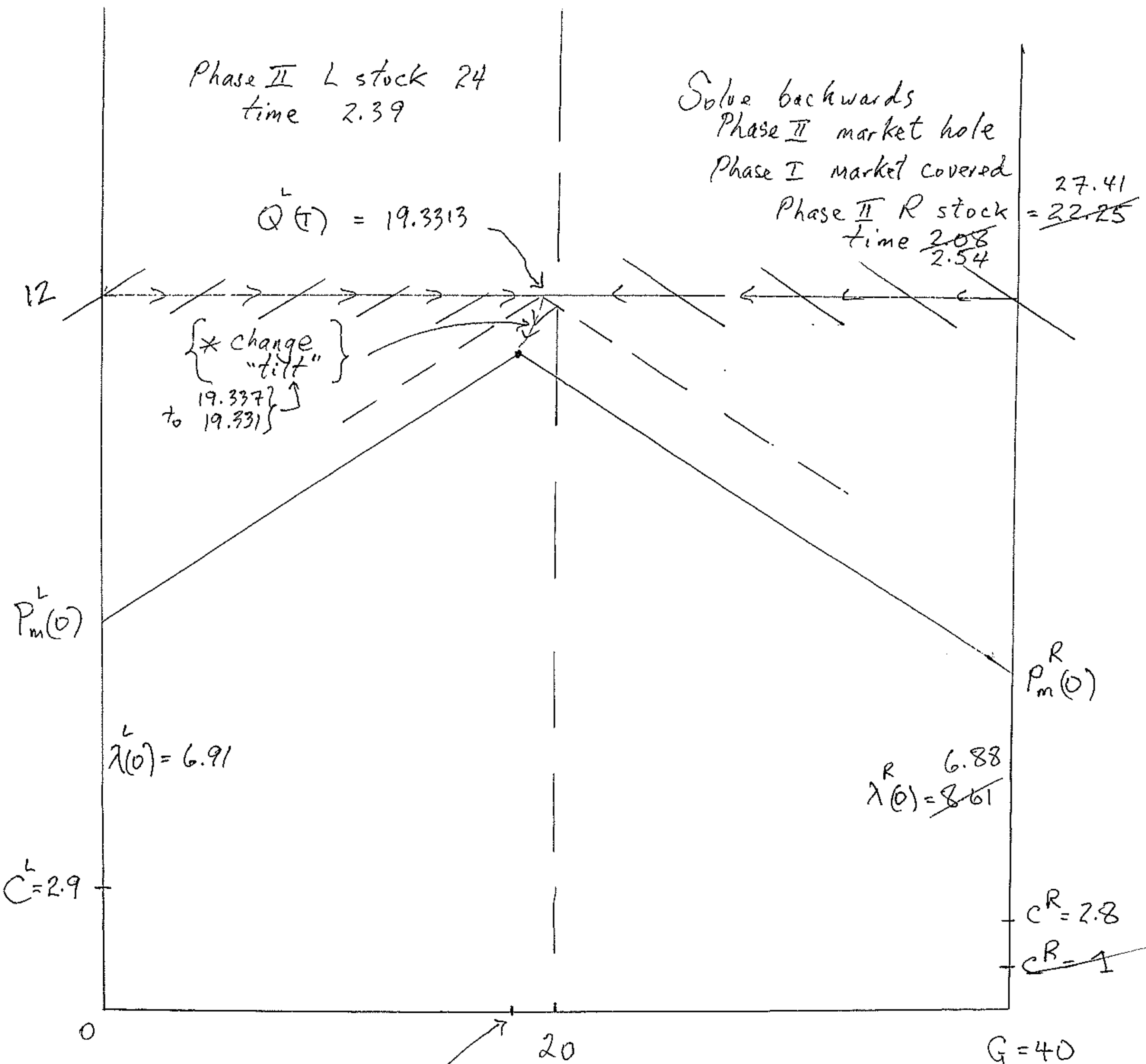

$Q^L(0) = \cancel{19.015}\ 19.337$

L uses up 7 units in Phase I

Time Phase I = ~~0.365~~ 0.362

R uses up ~~7.605~~ 7.48 units in Phase I

transport cost 0.1 per unit distance

discount rate = 0.1